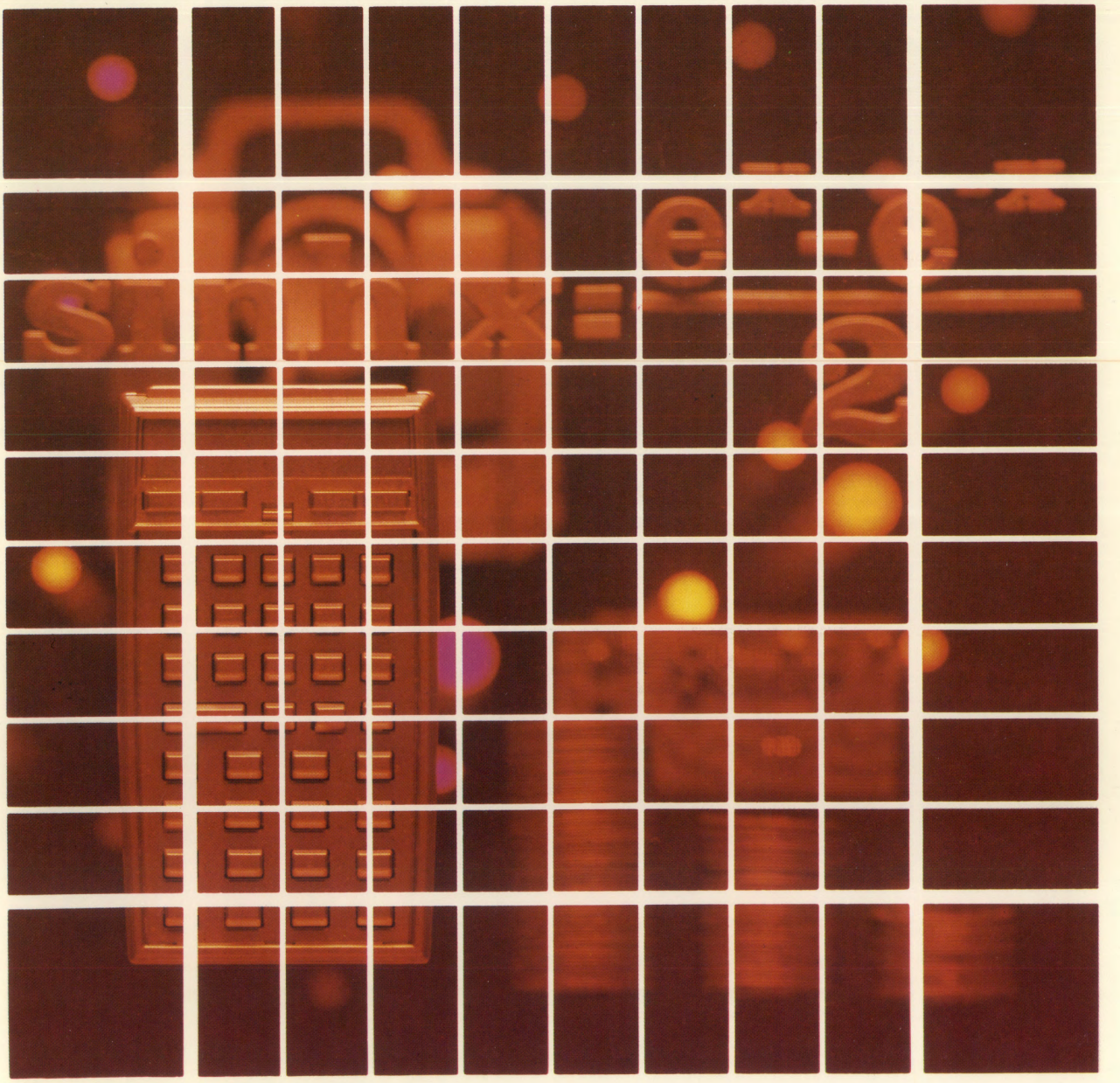
HEWLETT-PACKARD

HP-41C

USERS' LIBRARY SOLUTIONS

Geometry



NOTICE

The program material contained herein is supplied without representation or warranty of any kind. Hewlett-Packard Company therefore assumes no responsibility and shall have no liability, consequential or otherwise, of any kind arising from the use of this program material or any part thereof.

INTRODUCTION

This HP-41C Solutions book was written to help you get the most from your calculator. The programs were chosen to provide useful calculations for many of the common problems encountered.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software. The comments on each program listing describe the approach used to reach the solution and help you follow the programmer's logic as you become and expert on your HP calculator.

KEYING A PROGRAM INTO THE HP-41C

There are several things that you should keep in mind while you are keying in programs from the program listings provided in this book. The output from the HP 82143A printer provides a convenient way of listing and an easily understood method of keying in programs without showing every keystroke. This type of output is what appears in this handbook. Once you understand the procedure for keying programs in from the printed listings, you will find this method simple and fast. Here is the procedure:

1. At the end of each program listing is a listing of status information required to properly execute that program. Included is the SIZE allocation required. Before you begin keying in the program, press [XEQ] [ALPHA] SIZE [ALPHA] and specify the allocation (three digits; e.g., 10 should be specified as 010).

   Also included in the status information is the display format and status of flags important to the program. To ensure proper execution, check to see that the display status of the HP-41C is set as specified and check to see that all applicable flags are set or clear as specified.

2. Set the HP-41C to PRGM mode (press the [PRGM] key) and press [■] [GTO] [.] [.] to prepare the calculator for the new program.

3. Begin keying in the program. Following is a list of hints that will help you when you key in your programs from the program listings in this handbook.

   a. When you see " (quote marks) around a character or group of characters in the program listing, those characters are ALPHA. To key them in, simply press [ALPHA], key in the characters, then press [ALPHA] again. So "SAMPLE" would be keyed in as [ALPHA] "SAMPLE" [ALPHA].

   b. The diamond in front of each LBL instruction is only a visual aid to help you locate labels in the program listings. When you key in a program, ignore the diamond.

   c. The printer indication of divide sign is /. When you see / in the program listing, press [÷].

   d. The printer indication of the multiply sign is ⋇. When you see ⋇ in the program listing, press [×].

   e. The ⊢ character in the program listing is an indication of the [APPEND] function. When you see ⊢, press [■] [APPEND] in ALPHA mode (press [■] and the K key).

   f. All operations requiring register addresses accept those addresses in these forms:

      nn (a two-digit number)
      IND nn (INDIRECT: [■], followed fy a two-digit number)
      X, Y, Z, T, or L (a STACK address: [.] followed by X, Y, Z, T, or L)
      IND X, Y, Z, T or L (INDIRECT stack: [■] [.] followed by X, Y, Z, T, or L)

      Indirect addresses are specified by pressing [■] and then the indirect address. Stack addresses are specified by pressing [.] followed by X, Y, Z, T, or L. Indirect stack addresses are specified by pressing [■] [.] and X, Y, Z, T, or L.

Printer Listing	Keystrokes	Display
01◆LBL "SAMPLE"	[■] [LBL] [ALPHA] SAMPLE [ALPHA]	*01 LBL"SAMPLE*
02 "THIS IS A "	[ALPHA] *THIS IS A* [ALPHA]	*02"THIS IS A*
03 "⊢SAMPLE"	[ALPHA] [■] [APPEND] SAMPLE	*03" ⊢ SAMPLE*
04 AVIEW	[■] AVIEW [ALPHA]	*04 AVIEW*
05 6	6	*05 6*
06 ENTER↑	[ENTER↑]	*06 ENTER ↑*
07 -2	2 [CHS]	*07 -2*
08 /	[÷]	*08 /*
09 ABS	[XEQ] [ALPHA] ABS [ALPHA]	*09 ABS*
10 STO IND L	[STO] [■] [.] L	*10 STO IND L*
11 "R3="	[ALPHA] R3= [■] [ARCL] 03	*11"R3=*
12 ARCL 03	[■] [AVIEW]	*12 ARCL 03*
13 AVIEW	[ALPHA]	*13 AVIEW*
14 RTN	[■] [RTN]	*14 RTN*

TABLE OF CONTENTS

SINE PLATE SOLUTIONS, COORDINATE OF A POINT, POSITION AND SLOPE OF AN INCLINED HOLE

This program, with the aid of commonly available dowel pins and measuring tools, (and in the case of the sine plate, a sine plate and height blocks), will aid in accurately finding angles and heights for sine plates, position and slope of inclined holes and coordinates of points. All angular output is in decimal degrees.

Solution for Finding Coordinates of a Point:

Given: a, b, d and e, determine x and y

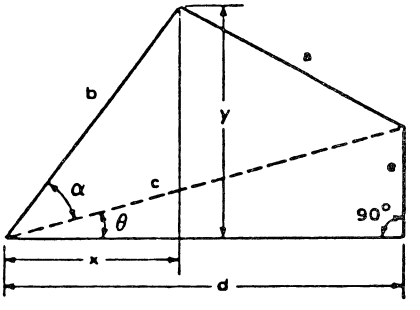
$$c = d^2 + e^2$$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\tan \theta = \frac{e}{d}$$

$$x = b \cos (\alpha + \theta)$$

$$y = b \sin (\alpha + \theta)$$

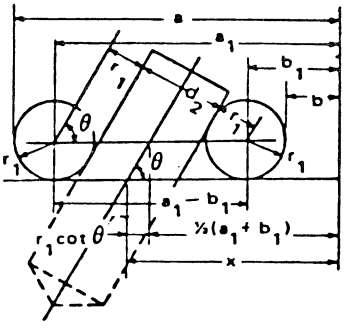


Solution for Finding the Location and Angle of an Inclined Hole:

Given: a, b, r_1, and d_2, determine θ and x

$$\sin \theta = \frac{2r_1 + d_2}{a_1 - b_1}$$

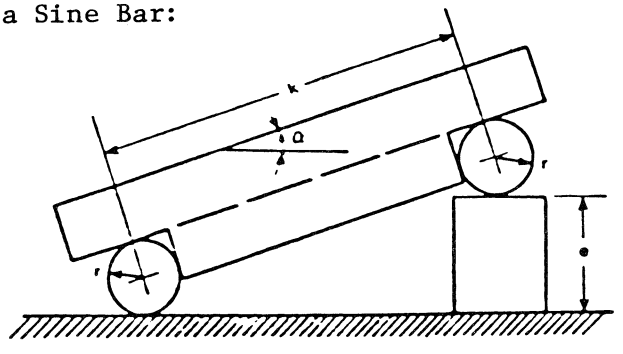
$$x = \tfrac{1}{2}(a_1 + b_1) + r_1 \cot \theta$$



Interchangeable Solutions for Work with a Sine Bar:

Given: e and k, determine α

$$\sin \alpha = \frac{e}{k}$$



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

Given: a = 1.630''
b = .260''
r_1 = .200''
d_2 = .4375''

Find θ, x of an inclined hole.

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 003	
[XEQ] [ALPHA] SINP [ALPHA]	SINE PLATE
[B]	R1 ?
.2 [R/S]	d2 ?
.4375 [R/S]	a?
1.63 [R/S]	b?
.26 [R/S]	THETA=59.7007
[R/S]	X=1.0619

User Instructions

SIZE: 003

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] SINP	SINE PLATE
3	To solve for the coordinates of a point:		[A]	a?
	Input a	a	[R/S]	b?
	b	b	[R/S]	d?
	d	d	[R/S]	e?
	and e.	e	[R/S]	X=(x)
			[R/S]	Y=(y)
4	To solve for the location and angle of an			
	inclined hole.		[B]	R1 ?
	Input r_1	r_1	[R/S]	d2 ?
	d_2	d_2	[R/S]	a?
	a	a	[R/S]	b?
	and b.	b	[R/S]	THETA=(θ)
			[R/S]	X=(x)
5	To solve for angles with a sine bar:		[C]	e?
	Input e	e	[R/S]	K?
	and K.	K	[R/S]	ALPHA=(α)
6	To solve for heights (of blocks) with a			
	sine bar:		[D]	K?
	Input K	K	[R/S]	ALPHA?
	and α.	α	[R/S]	e=(e)

Program Listings

```
 01*LBL "SIN
P"
 02 "SINE PL
ATE"
 03 AVIEW
 04 STOP
 05*LBL A
 06 "a?"
 07 PROMPT
 08 X↑2
 09 "b?"
 10 PROMPT
 11 STO 00
 12 X↑2
 13 -
 14 "d?"
 15 PROMPT
 16 "e?"
 17 PROMPT
 18 X<>Y
 19 R-P
 20 STO 01
 21 X<>Y
 22 STO 02
 23 RDN
 24 X↑2
 25 -
 26 CHS
 27 RCL 00
 28 /
 29 RCL 01
 30 /
 31 2
 32 /
 33 ACOS
 34 RCL 02
 35 +
 36 RCL 00
 37 P-R
 38 "X"
 39 XEQ 11
 40 RDN
 41 "Y"
 42 XEQ 11
 43*LBL B
 44 "R1 ?"
 45 PROMPT
 46 STO 00
 47 2
 48 *
 49 "d2 ?"
 50 PROMPT
 51 +
 52 "a?"
 53 PROMPT
 54 RCL 00
 55 -
 56 STO 01
 57 "b?"
 58 PROMPT
 59 RCL 00
 60 +
 61 ST+ 01
 62 -
 63 /
 64 ASIN
 65 "THETA"
 66 XEQ 11
 67 TAN
 68 1/X
 69 RCL 00
 70 *
 71 RCL 01
 72 2
 73 /
 74 +
 75 "X"
 76 XEQ 11
 77*LBL C
 78 "e?"
 79 PROMPT
 80 "K?"
 81 PROMPT
 82 /
 83 ASIN
 84 "ALPHA"
 85 XEQ 11
 86*LBL D
 87 "K?"
 88 PROMPT
 89 "ALPHA?"
 90 PROMPT
 91 SIN
 92 *
 93 "ε"
 94*LBL 11
 95 "⊢="
 96 ARCL X
 97 AVIEW
 98 STOP
 99 RTN
100 .END.
```

Initialize

Input a, b, d, and e

Calculate x,y

Input r_1, d_2, a, b

Calculate θ, x

Input e, k

Calculate α

Input α, k

Calculate e

Display routine

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	b or r_1	50	
	c or a_1, $a_1 + b_1$		
	θ		
05		55	
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 003 TOT. REG. 29 USER MODE
ENG ____ FIX ____ SCI ____ ON X OFF ____
DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

V NOTCHES AND LONG RADII

This program, together with commonly available dowel pins and height gages, will accurately determine the position and angles of "V" grooves or notches. With the same tools, long radii are accurately measured. All angular output is in decimal degrees.

Given: a, b, c, d, r_1 and r_2, determine x, y, α and β:

$$\tan \phi = \frac{b_1 - a_1}{d_1 - c_1}$$

$$\overline{O_1O_2} = \frac{d_1 - c_1}{\cos \phi}$$

$$\sin \theta = \frac{r_2 - r_1}{\overline{O_1O_2}}$$

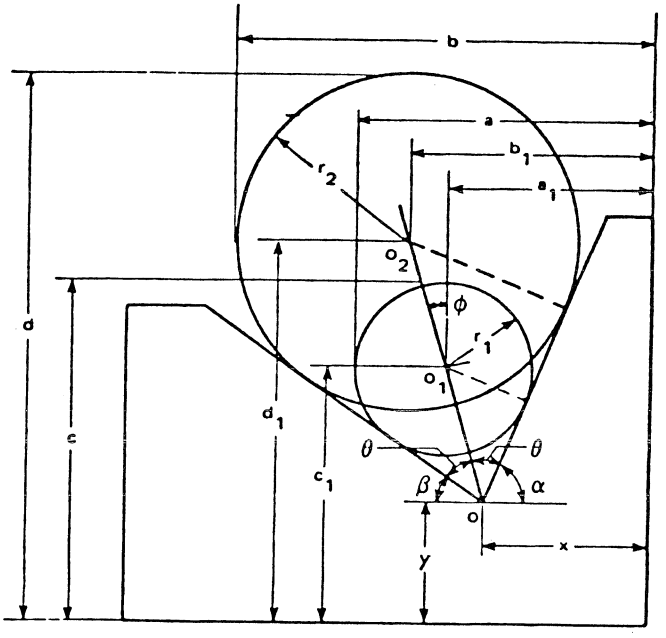
$$\overline{OO_1} = \frac{r_1}{\sin \theta}$$

$$x = a_1 - \overline{OO_1} \sin \phi$$

$$y = c_1 - \overline{OO_1} \cos \phi$$

$$\alpha = 90° + \phi - \theta$$

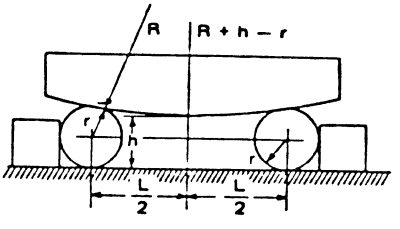
$$\beta = 90° - \phi - \theta$$



Given: L, r and h, determine R:

$$(R + r)^2 = (R + h - r)^2 + (\tfrac{1}{2}) L^2$$

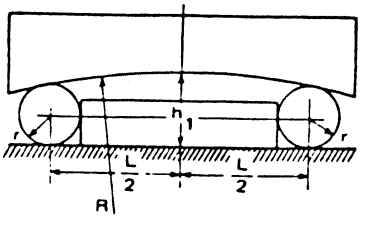
$$R = \frac{L^2}{8(2r - h)} - \frac{h}{2}$$



Given L, r and h , determine R:

$$(R - r)^2 = (R - h_1 + r)^2 + (\tfrac{1}{2} L)^2$$

$$R = \frac{L^2}{8(h_1 - 2r)} + \frac{h_1}{2}$$



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example 1:

For Long Radius (concave arc)

L = 1.000''
r = .15625''
h = .270''

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 006	
[XEQ] [ALPHA] VNOTCH [ALPHA]	V NOTCHES, L.R.
[B]	L?
1 [R/S]	R?
.15625 [R/S]	H?
.27 [R/S]	R=2.8062

Example 2:

For "V" Notch

a = 1.500'' d = 2.800''
b = 2.125'' r_1 = .4375''
c = 1.750'' r_2 = .875''

Keystrokes:	Display:
[A]	R2?
.875 [R/S]	R1?
.4375 [R/S]	a?
1.5 [R/S]	b?
2.125 [R/S]	c?
1.75 [R/S]	d?
2.8 [R/S]	X=0.8750
[R/S]	Y=0.7000
[R/S]	ALPHA=63.9420
[R/S]	BETA=29.9010

User Instructions

SIZE: 006

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] VNOTCH	V NOTCHES, L.R.
3	To solve for "V" Notch:		[A]	R2?
	Input r_2	r_2	[R/S]	R1?
	r_1	r_1	[R/S]	a?
	a	a	[R/S]	b?
	b	b	[R/S]	c?
	c	c	[R/S]	d?
	and d and calculate X	d	[R/S]	X=(x)
	y		[R/S]	Y=(y)
	α		[R/S]	ALPHA=(α)
	and β.		[R/S]	BETA=(β)
4	To solve for long radii:			
4a	(concave radius)		[C]	L?
	Input L	L	[R/S]	R?
	r	r	[R/S]	H?
	h and calculate R.	h	[R/S]	R=(R)
4b	(convex radius)		[B]	L?
	Input L	L	[R/S]	R?
	r	r	[R/S]	H?
	h and calculate R	h	[R/S]	R=(R)

Program Listings

```
 01*LBL "VNO
TCH"
 02 CF 00
 03 "V NOTCH
ES, L.R."
 04 AVIEW
 05 STOP
 06*LBL C
 07 SF 00
 08*LBL B
 09 "L?"
 10 PROMPT
 11 X↑2
 12 "R?"
 13 PROMPT
 14 2
 15 *
 16 FS? 00
 17 CHS
 18 "H?"
 19 PROMPT
 20 FS?C 00
 21 CHS
 22 STO 00
 23 -
 24 8
 25 *
 26 /
 27 RCL 00
 28 2
 29 /
 30 -
 31 "R"
 32*LBL 11
 33 "⊢="
 34 ARCL X
 35 AVIEW
 36 STOP
 37 RTN
 38*LBL A
 39 "R2?"
 40 PROMPT
 41 STO 00
 42 "R1?"
 43 PROMPT
 44 STO 01
 45 -
 46 "a?"
 47 PROMPT
 48 LASTX
 49 -
 50 STO 02
 51 "b?"
 52 PROMPT
 53 RCL 00
 54 -
 55 -
 56 "c?"
 57 PROMPT
 58 RCL 01
 59 -
 60 STO 03
 61 "d?"
 62 PROMPT
 63 RCL 00
 64 -
 65 -
 66 STO 05
 67 /
 68 ATAN
 69 STO 04
 70 CLX
 71 RCL 05
 72 CHS
 73 RCL 04
 74 COS
 75 /
 76 RCL 00
 77 RCL 01
 78 -
 79 /
 80 1/X
 81 ASIN
 82 STO 05
 83 RCL 04
 84 RCL 01
 85 LASTX
 86 /
 87 P-R
 88 ST- 03
 89 RDN
 90 ST- 02
 91 RCL 04
 92 90
 93 RCL 05
 94 -
 95 +
 96 LASTX
 97 RCL 04
 98 -
 99 "X"
100 RCL 02
```

Initialize

Concave arcs

Input L, r, h

Calculate R

Display routine

Input a, b, c, d, r_1, and r_2

Calculate x, y, α, β

Display results

Program Listings

```
101 XEQ 11
102 "Y"
103 RCL 03
104 XEQ 11
105 "ALPHA"
106 RCL T
107 XEQ 11
108 "BETA"
109 RCL T
110 XEQ 11
111 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

#		#	
00	r_2 or $\pm h$	50	
	r_1		
	a_1, x		
	c_1, y		
	ϕ		
05	$c_1 - d_1$, θ	55	
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 006 TOT. REG. 33 USER MODE

ENG ___ FIX ___ SCI ___ ON X OFF ___

DEG X RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
00		Concave arc	Convex arc

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

INTERNAL AND EXTERNAL TAPERS

This program, used with commonly available dowel pins, height bases, and balls, will accurately determine the position and angle of both external and internal tapers. All angular output is in decimal degrees.

Internal Taper:

Given b, c, d, r_1 and r_2, determine C, D, ϕ and R_1

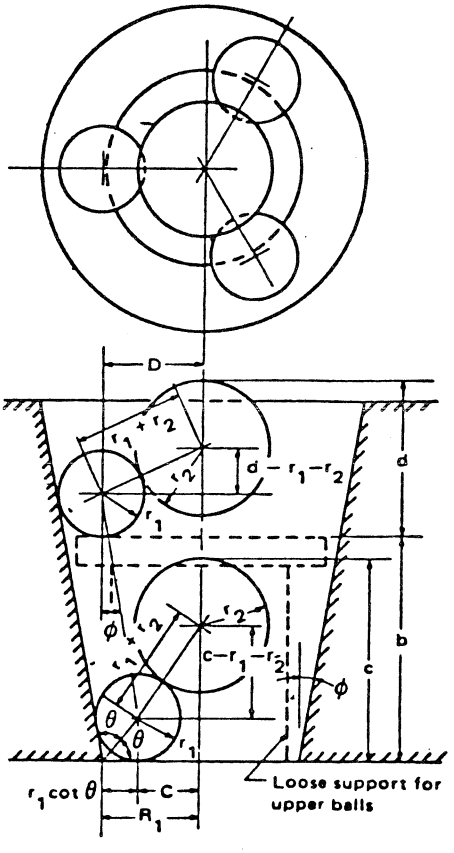
$$C^2 = 2c(r_1 + r_2) - c^2$$

$$D^2 = 2d(r_1 + r_2) - d^2$$

$$\tan \phi = \frac{D - C}{b}$$

$$2\theta = 90° + \phi$$

$$R_1 = C + r_1 \cot \theta$$



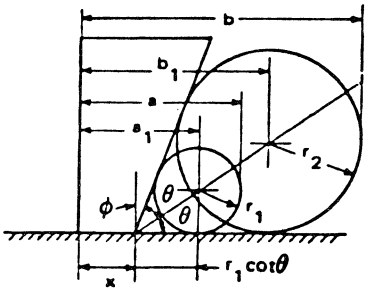
External Taper Case #1:

Given: a, b, r_1 and r_2, determine x and ϕ

$$\tan \theta = \frac{r_2 - r_1}{b_1 - a_1}$$

$$\phi = 90° - 2\theta$$

$$x = a_1 - r_1 \cot \theta$$



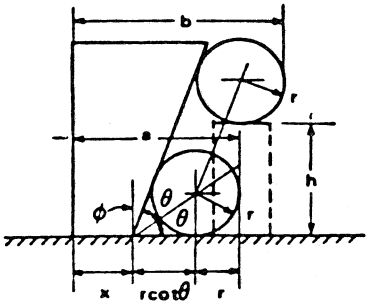
External Taper Case #2:

Given a, b, r and h, determine x and ϕ

$$\tan 2\theta = \frac{h}{b - a}$$

$$\phi = 90 - 2\theta$$

$$x = a - r - r\cot\theta$$

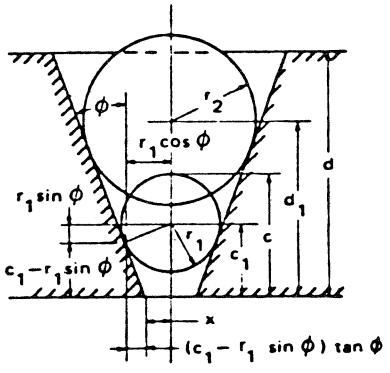


External Taper Case #3:

Given c, d, r_1 and r_2, determine x and ϕ

$$\sin\phi = \frac{r_2 - r_1}{d_1 - c_1}$$

$$x = \frac{r_1}{\cos\phi} - c_1 \tan\phi$$



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

Internal Taper: Given b = 1.150'' r_1 = .21875''
c = 1.050'' r_2 = .34375''
d = .800''

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 005	
[XEQ] [ALPHA] TAPERS [ALPHA]	IN. ,EX. TAPERS
[A]	R1?
.21875 [R/S]	R2?
.34375 [R/S]	c?
1.05 [R/S]	d?
.8 [R/S]	b?
1.15 [R/S]	C=0.2806
[R/S]	D=0.5099
[R/S]	PHI=11.2753 (degs)
[R/S]	R1=0.4601

User Instructions

SIZE: 005

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] TAPERS	IN. ,EX. TAPERS
3	Determine the case from the drawings.			
4	For internal taper:		[A]	R1?
		r_1	[R/S]	R2?
		r_2	[R/S]	c?
		c	[R/S]	d?
		d	[R/S]	b?
		b	[R/S]	C=(c)
			[R/S]	D=(d)
			[R/S]	PHI=(ϕ)
			[R/S]	R1=(R1)
5	For an external taper, case 1:		[B]	R1?
		r_1	[R/S]	R2?
		r_2	[R/S]	a?
		a	[R/S]	b?
		b	[R/S]	X=(x)
			[R/S]	PHI=(ϕ)
6	For an external taper, case 2:		[C]	H?
		h	[R/S]	b?
		b	[R/S]	a?
		a	[R/S]	R?
		r	[R/S]	X=(x)
			[R/S]	PHI=(ϕ)
7	For an external taper, case 3:		[D]	R1?
		r_1	[R/S]	R2?
		r_2	[R/S]	c?

User Instructions

SIZE: 005

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
		c	[R/S]	d?
		d	[R/S]	X=(x)
			[R/S]	PHI=(ϕ)

Program Listings

```
 01*LBL "TAP
ERS"
 02 CF 01
 03 "IN./EX.
 TAPERS"
 04 AVIEW
 05 STOP
 06*LBL A
 07 "R1?"
 08 PROMPT
 09 STO 00
 10 "R2?"
 11 PROMPT
 12 +
 13 2
 14 *
 15 STO 01
 16 "c?"
 17 PROMPT
 18 *
 19 LASTX
 20 X↑2
 21 -
 22 SQRT
 23 STO 02
 24 RCL 01
 25 "d?"
 26 PROMPT
 27 *
 28 LASTX
 29 X↑2
 30 -
 31 SQRT
 32 STO 04
 33 RCL 02
 34 -
 35 "b?"
 36 PROMPT
 37 /
 38 ATAN
 39 STO 03
 40 90
 41 +
 42 2
 43 /
 44 TAN
 45 1/X
 46 ST* 00
 47 RCL 02
 48 "C"
 49 XEQ 11
```

Initialize

Internal taper

Input r_1, r_2, c, d and b

Calculate C, D, ϕ, R1

```
 50 RCL 04
 51 "D"
 52 XEQ 11
 53 RCL 03
 54 "PHI"
 55 XEQ 11
 56 RCL 02
 57 RCL 00
 58 +
 59 "R1"
 60 XEQ 11
 61*LBL B
 62 "R1?"
 63 PROMPT
 64 STO 00
 65 "R2?"
 66 PROMPT
 67 STO 01
 68 -
 69 "a?"
 70 FS? 01
 71 "c?"
 72 PROMPT
 73 RCL 00
 74 -
 75 STO 02
 76 "b?"
 77 FS? 01
 78 "d?"
 79 PROMPT
 80 RCL 01
 81 -
 82 -
 83 /
 84 FS?C 01
 85 RTN
 86 ATAN
 87 STO 03
 88 LASTX
 89 1/X
 90 RCL 00
 91 *
 92 RCL 02
 93 -
 94 CHS
 95 90
 96 RCL 03
 97 2
 98 *
 99 -
100 X<>Y
```

External taper case 1

Input r_1, r_2, a and b

Calculate x, ϕ

Program Listings

```
101 GTO 05
102*LBL C
103 "H?"
104 PROMPT
105 "b?"
106 PROMPT
107 "a?"
108 PROMPT
109 STO 00
110 -
111 /
112 ATAN
113 STO 01
114 CHS
115 90
116 +
117 RCL 00
118 RCL 01
119 2
120 /
121 TAN
122 1/X
123 1
124 +
125 "R?"
126 PROMPT
127 *
128 -
129*LBL 05
130 "X"
131 XEQ 11
132 RDN
133 "PHI"
134 XEQ 11
135*LBL D
136 SF 01
137 XEQ B
138 ASIN
139 STO 03
140 RCL 00
141 LASTX
142 RCL 02
143 *
144 -
145 RCL 03
146 COS
147 /
148 GTO 05
149*LBL 11
150 "⊢="
151 ARCL X
152 AVIEW
153 STOP
154 RTN
155 .END.
```

External taper, case 2

Input h, b, a and r

Calculate x, ϕ

Display x, ϕ

External taper, case 3

Input r_1, r_2, c and d

Calculate x, ϕ

Display routine

60	
70	
80	
90	
00	

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	used	50	
	used		
	used		
	used		
	used		
05		55	
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 005 TOT. REG. 41 USER MODE

ENG ____ FIX ____ SCI ____ ON X OFF ____

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
01	C	Case 3	Case 1

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

POINTS OF TANGENCY WITH CIRCLES AND ARCS

This program will accurately locate points of tangency between straight lines and arcs, between straight lines and a circle, and between two circles and a straight line. All angular outputs are in decimal degrees.

Solutions for Finding Point of Tangency With an Arc:

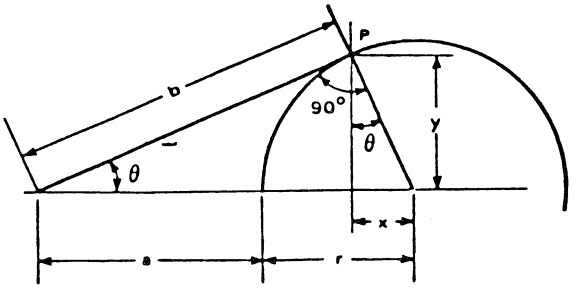
Given: a and r, determine x and y

$$b^2 = (a + r)^2 - r^2$$

$$\sin\theta = \frac{r}{a + r} = \frac{y}{b} = \frac{x}{r}$$

$$x = \frac{r^2}{a + r}$$

$$y = \frac{br}{a + r}$$



Solution for Finding Points of Tangency with A Circle:

Given: b, c and r, determine x_1 and y_1

$$a = \sqrt{b^2 + c^2} - r$$

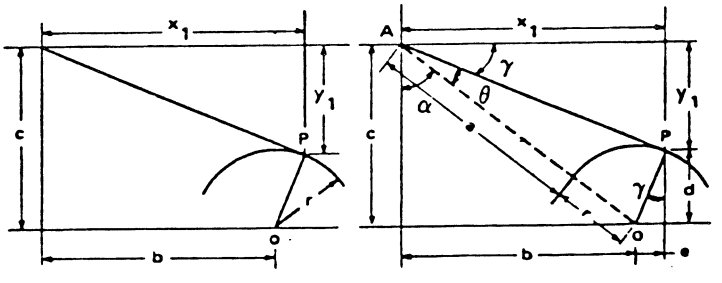
$$\sin\theta = \frac{r}{a + r}$$

$$\tan\alpha = \frac{b}{c}$$

$$\gamma = 90° - \theta - \alpha$$

$$e = r\sin\gamma, \text{ then } x_1 = b + e$$

$$d = r\cos\gamma, \text{ then } y_1 = c - d$$



Solution for Finding Points of Tangency with Two Circles:

Given: a, b, r_1 and r_2, determine x_1, y_1, x_2 and y_2

$$c = a^2 + b^2$$

$$\tan \theta = \frac{b}{a}$$

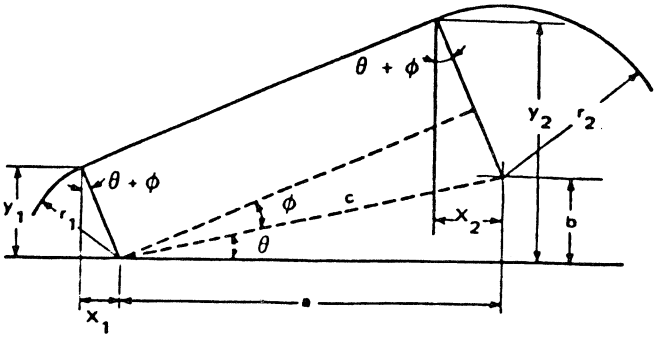
$$\sin \phi = \frac{r_2 - r_1}{c}$$

$$x_1 = r_1 \sin(\theta + \phi)$$

$$y_1 = r_1 \cos(\theta + \phi)$$

$$x_2 = r_2 \sin(\theta + \phi)$$

$$y_2 = b + r_2 \cos(\theta + \phi)$$



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

Find the point of tangency with an arc given: a = 1.125'' and r = .750''.

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 007	
[XEQ] [ALPHA] POINTS [ALPHA]	POINTS OF T.
[A]	a?
1.125 [R/S]	R?
.75 [R/S]	X=0.3000
[R/S]	Y=0.6874
[R/S]	b=1.7185
[R/S]	THETA=23.5782 (degs)

User Instructions

SIZE: 007

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] POINTS	POINTS OF T.
3	Determine the case from the drawings.			
4	For the point of tangency with an arc:		[A]	a?
		a	[R/S]	R?
		r	[R/S]	X=(x)
			[R/S]	Y=(y)
	(optional)		[R/S]	b=(b)
			[R/S]	THETA=(θ)
5	For the point of tangency with a circle:		[B]	b?
		b	[R/S]	c?
		c	[R/S]	R?
		r	[R/S]	X1=(x_1)
			[R/S]	Y1=(y_1)
	(optional)		[R/S]	a=(a)
			[R/S]	THETA=(θ)
			[R/S]	ALPHA=(α)
6	For the points of tangency with two			
	circles:		[C]	a?
		a	[R/S]	b?
		b	[R/S]	R2?
		r_2	[R/S]	R1?
		r_1	[R/S]	X1=(x_1)
			[R/S]	Y1=(y_1)
			[R/S]	X2=(x_2)
			[R/S]	Y2=(y_2)
	(optional)		[R/S]	c=(c)

User Instructions

SIZE: 007

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
			[R/S]	THETA=(θ)
			[R/S]	PHI=(ϕ)

Program Listings

```
 01*LBL "POI
NTS"
 02 "POINTS
OF T."
 03 AVIEW
 04 STOP
 05*LBL A
 06 "a?"
 07 PROMPT
 08 "R?"
 09 PROMPT
 10 STO 01
 11 +
 12 STO 00
 13 X↑2
 14 RCL 01
 15 X↑2
 16 -
 17 SQRT
 18 STO 02
 19 RCL 01
 20 RCL 00
 21 /
 22 *
 23 LASTX
 24 RCL 01
 25 *
 26 "X"
 27 XEQ 11
 28 RDN
 29 "Y"
 30 XEQ 11
 31 RCL 02
 32 "b"
 33 XEQ 11
 34 /
 35 ASIN
 36 "THETA"
 37 XEQ 11
 38*LBL B
 39 "b?"
 40 PROMPT
 41 STO 00
 42 "c?"
 43 PROMPT
 44 STO 01
 45 R-P
 46 "R?"
 47 PROMPT
 48 STO 04
 49 -
 50 STO 02
 51 LASTX
 52 LASTX
 53 RCL 02
 54 +
 55 /
 56 ASIN
 57 STO 03
 58 90
 59 -
 60 CHS
 61 RCL 00
 62 RCL 01
 63 /
 64 ATAN
 65 STO 05
 66 -
 67 RCL 04
 68 P-R
 69 RCL 01
 70 -
 71 CHS
 72 X<>Y
 73 RCL 00
 74 +
 75 "X1"
 76 XEQ 11
 77 RDN
 78 "Y1"
 79 XEQ 11
 80 RCL 02
 81 "a"
 82 XEQ 11
 83 RCL 03
 84 "THETA"
 85 XEQ 11
 86 RCL 05
 87 "ALPHA"
 88 XEQ 11
 89*LBL C
 90 "a?"
 91 PROMPT
 92 STO 00
 93 "b?"
 94 PROMPT
 95 STO 01
 96 R-P
 97 STO 02
 98 "R2?"
 99 PROMPT
100 STO 04
```

Initialize

Tangency with an arc

Input a, r

Calculate x, y, b, θ

Tangency with a circle

Input b, c, r

Calculate x_1, y_1, a, θ, α

Tangency with two circles

Program Listings

```
101 "R1?"
102 PROMPT
103 STO 05
104 -
105 /
106 1/X
107 ASIN
108 STO 03
109 RCL 01
110 RCL 00
111 /
112 ATAN
113 STO 00
114 +
115 STO 06
116 RCL 05
117 P-R
118 X<>Y
119 "X1"
120 XEQ 11
121 X<>Y
122 "Y1"
123 XEQ 11
124 RCL 06
125 RCL 04
126 P-R
127 RCL 01
128 +
129 X<>Y
130 "X2"
131 XEQ 11
132 X<>Y
133 "Y2"
134 XEQ 11
135 RCL 02
136 "c"
137 XEQ 11
138 RCL 00
139 "THETA"
140 XEQ 11
141 RCL 03
142 "PHI"
143◆LBL 11
144 "⊢="
145 ARCL X
146 AVIEW
147 STOP
148 RTN
149 .END.
```

Input a, b, r_1, r_2

Calculate x_1, y_1, x_2, y_2, c, θ, ϕ

Display routine

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	used	50	
	used		
	used		
	used		
	used		
05	used	55	
	used		
	used		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 007 TOT. REG. 45 USER MODE

ENG ____ FIX ____ SCI ____ ON X OFF ____

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

LINE-LINE INTERSECTION

This program will calculate the point of intersection of two lines. For each line the user specifies two points, or one point and the angle from horizontal, or one point and the slope. Slope will be converted to angle by the relation $\theta = \tan^{-1}(\text{slope})$. Given two points (x_1,y_1) and (x_2,y_2) on the line, the angle is:

$$\theta = \tan^{-1} \frac{y_2 - y_1}{x_2 - x_1}$$

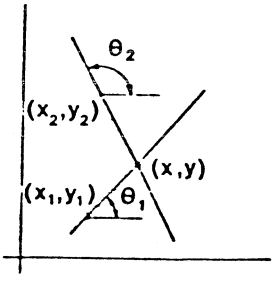
(x,y) = Coordinates of point of intersection

(x_1,y_1) = Coordinates of point on line one

(x_2,y_2) = Coordinates of point on line two

θ_1 = Angle from horizontal to line one

θ_2 = Angle from horizontal to line two



Equations:

$$x = \frac{x_1 \tan\theta_1 - x_2 \tan\theta_2 + y_2 - y_1}{\tan\theta_1 - \tan\theta_2}$$

$$y = y_1 + (x - x_1)\tan\theta_1$$

Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

Find the point of intersection of two lines, one passing through (10,20) (40,30), and the other through (-10,30) (50,10).

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 007	
[XEQ] [ALPHA] LINE [ALPHA]	LINE INTRSEC
[A]	X1 ?
10 [R/S]	Y1 ?
20 [R/S]	X2 ?
40 [R/S]	Y2 ?
30 [R/S]	NEXT LINE ?
[A]	X1 ?
10 [CHS] [R/S]	Y1 ?
30 [R/S]	X2 ?
50 [R/S]	Y2 ?
10 [R/S]	X=15.0000
[R/S]	Y=21.6667

User Instructions

SIZE: 007

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] LINE	LINE INTRSEC
	LINE-LINE INTERSECTION:			
3a	Input two points on line:		[A]	X1 ?
		x_1	[R/S]	Y1 ?
		y_1	[R/S]	X2 ?
		x_2	[R/S]	Y2 ?
		y_2	[R/S]	NEXT LINE ?
3b	Or, input one point and the slope, m:		[B]	X ?
		x	[R/S]	Y ?
		y	[R/S]	M ?
		m	[R/S]	NEXT LINE ?
3c	Or, input one point and the angle θ:		[C]	X ?
		x	[R/S]	Y ?
		y	[R/S]	THETA ?
		θ	[R/S]	NEXT LINE ?
3d	Or, for the case where the second line is			
	vertical, input the x coordinate:	x	[D]	Y=(y)
4	Repeat step 3 for the second line.			
5	After the parameters for the second line			
	are input, the intersection coordinates			X=(x)
	are automatically displayed.		[R/S]	Y=(y)
6	For a new case, go to step 2.			

Program Listings

```
 01*LBL "LIN
E"
 02 CF 00
 03 1.006
 04 STO 00
 05 "LINE IN
TRSEC"
 06 AVIEW
 07 STOP
 08*LBL 01
 09 AVIEW
 10 STOP
 11*LBL A
 12 "X1 ?"
 13 PROMPT
 14 STO IND
00
 15 ISG 00
 16 "Y1 ?"
 17 PROMPT
 18 STO IND
00
 19 X<>Y
 20 "X2 ?"
 21 PROMPT
 22 -
 23 X<>Y
 24 "Y2 ?"
 25 PROMPT
 26 -
 27 /
 28 1/X
 29 GTO a
 30*LBL C
 31 SF 00
 32*LBL B
 33 "X ?"
 34 PROMPT
 35 STO IND
00
 36 ISG 00
 37 "Y ?"
 38 PROMPT
 39 STO IND
00
 40 "M ?"
 41 FS? 00
 42 "THETA ?
"
 43 PROMPT
 44 FS?C 00
```

Initialize

Input x_1, y_1 and x_2, y_2

Calculate θ

Input x, y and slope or θ

```
 45 TAN
 46*LBL a
 47 ISG 00
 48 STO IND
00
 49 "NEXT LI
NE ?"
 50 ISG 00
 51 GTO 01
 52*LBL E
 53 RCL 01
 54 RCL 03
 55 *
 56 RCL 04
 57 RCL 06
 58 *
 59 -
 60 RCL 05
 61 +
 62 RCL 02
 63 -
 64 RCL 03
 65 RCL 06
 66 -
 67 /
 68 "X"
 69 XEQ 11
 70*LBL D
 71 RCL 01
 72 -
 73 RCL 03
 74 *
 75 RCL 02
 76 +
 77 "Y"
 78*LBL 11
 79 "⊢="
 80 ARCL X
 81 AVIEW
 82 STOP
 83 RTN
 84 .END.
```

Calculate x, y

Display routine

00

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	used	50	
	used		
	used		
	used		
	used		
05	used	55	
	used		
	used		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 007 TOT. REG. 32 USER MODE
ENG ____ FIX ____ SCI ____ ON X OFF ____
DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

POINTS ON A STRAIGHT LINE

This program calculates the coordinates of equidistant points on a straight line.

Equations:

Point P_i is calculated by

$$x_i = x_1 + (i - 1)\,H \cos\theta$$

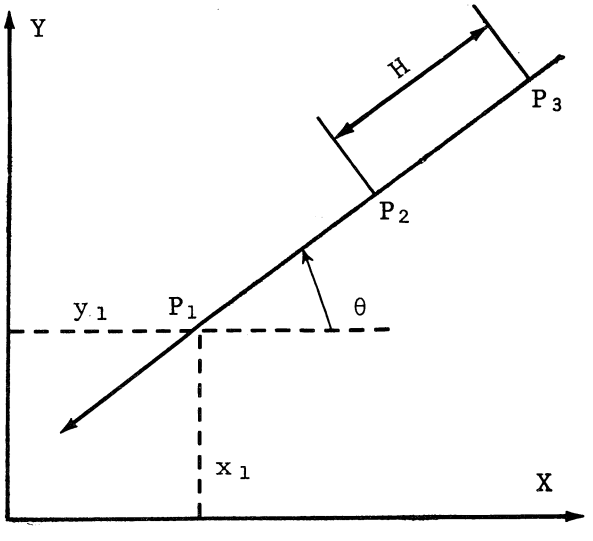
$$y_i = y_1 + (i - 1)\,H \sin\theta, \quad i = \pm 0, 1, 2, \ldots$$

where

$P_1 = (x_1, y_1)$ (the starting point);

θ is the angle of the straight line with the x axis;

H is the distance between consecutive points in the direction of the straight line.



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

For the straight line designated by $X_1 = 10$, $Y_1 = 10$, $\theta = -30°$, calculate P_i for $H = 20$ and $i = 1, 2,$ and 3.

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 005	
[XEQ] [ALPHA] PLINE [ALPHA]	PTS. ON ST. L.
	X1 ?
10 [R/S]	Y1 ?
10 [R/S]	THETA ?
30 [CHS] [R/S]	H ?
20 [R/S]	I ?
1 [R/S]	X=10.0000
[R/S]	Y=10.0000
[R/S] [R/S]	X=27.3205
[R/S]	Y=0.0000
[R/S] [R/S]	X=44.6410
[R/S]	Y=-10.0000

User Instructions

SIZE: 005

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] PLINE	PTS. ON ST. L.
				X1 ?
3	Input P_1, the angle of the line, and the	x_1	[R/S]	Y1 ?
	distance between points.	y_1	[R/S]	THETA ?
		θ	[R/S]	H ?
		H	[R/S]	I ?
4	Calculate P_i.	i	[R/S]	X=(x)
			[R/S]	Y=(y)
5a	To calculate a new P_j,		[R/S]	I ?
	and go to step 4, or			
5b	To calculate $P_{(i+1)}$		[R/S]	I ?
			[R/S]	X=(x)
			[R/S]	Y=(y)

Program Listings

```
 01*LBL "PLI
NE"
 02 "PTS. ON
 ST. L."
 03 AVIEW
 04 PSE
 05 "X1 ?"
 06 PROMPT
 07 STO 02
 08 "Y1 ?"
 09 PROMPT
 10 STO 03
 11 "THETA ?
"
 12 PROMPT
 13 "H ?"
 14 PROMPT
 15 P-R
 16 STO 00
 17 RDN
 18 STO 01
 19*LBL 01
 20 "I ?"
 21 PROMPT
 22 1
 23 -
 24 STO 04
 25 RCL 00
 26 *
 27 RCL 02
 28 +
 29 "X"
 30 XEQ 11
 31 RCL 04
 32 RCL 01
 33 *
 34 RCL 03
 35 +
 36 "Y"
 37 XEQ 11
 38 RCL 04
 39 2
 40 +
 41 GTO 01
 42*LBL 11
 43 "⊢="
 44 ARCL X
 45 AVIEW
 46 STOP
 47 RTN
 48 .END.
```

Initialize

Input x_1, y_1, θ, and H

Input I

Calculate x, y

Display routine

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	h cos	50	
	h sin		
	x_1		
	y_1		
	i+1		
05		55	
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 005 TOT. REG. 19 USER MODE

ENG ____ FIX ____ SCI ____ ON X OFF ____

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

GRID OF POINTS: CALCULATE ALL POINTS

This program calculates the X and Y coordinates of all the points on a grid defined as follows:

a. First direction of a grid:

the angle, θ_1, with the positive X axis

the algebraic distance between each point, H_1, in this direction

the total number, N_1, of points (including the first one)

b. Second direction of the grid:

the angle, θ_2, with the positive X axis

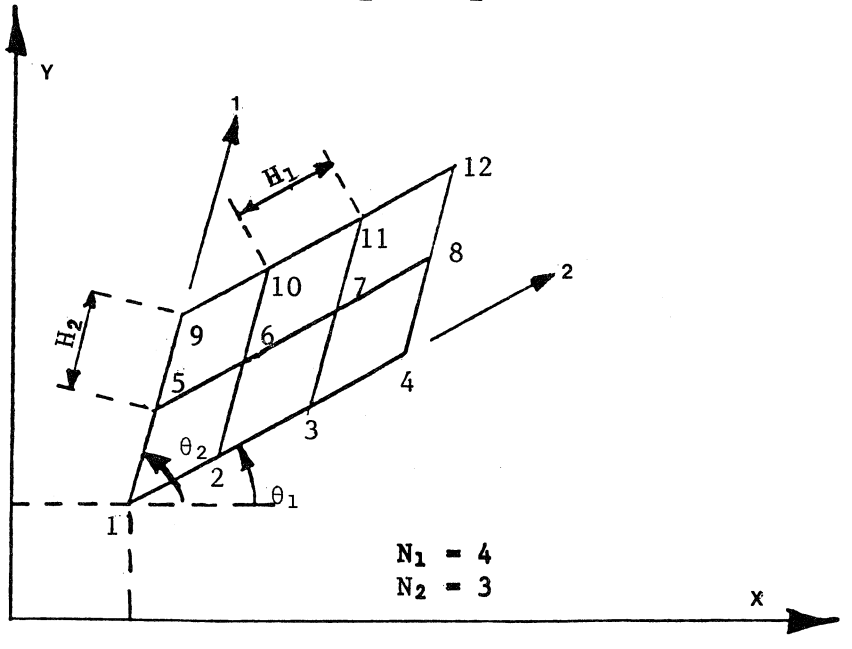
the algebraic distance between two points, H_2, in that direction

the total number, N_2, of points (including the first one)

c. Starting point (noted 1) with coordinates X and Y.

The calculation is incremental from point 1 to point (N_1,N_2). For each point we find:

The index i, the X_i and Y_i coordinates



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

Find the grid points for:

$\theta_1 = 0°$, $H_1 = 10$, $N_1 = 3$, $X_1 = 10$

$\theta_2 = 90°$, $H_2 = 20$, $N_2 = 2$, $Y_2 = 10$

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 010	
[XEQ] [ALPHA] GRIDALL [ALPHA]	GRID ALL PTS
	N1 ?
3 [R/S]	N2 ?
2 [R/S]	X1 ?
10 [R/S]	Y1 ?
10 [R/S]	H1 ?
10 [R/S]	H2 ?
20 [R/S]	THETA 1 ?
0 [R/S]	THETA 2 ?
90 [R/S]	X1=10.0000
[R/S]	Y1=10.0000
[R/S]	X2=20.0000
[R/S]	Y2=10.0000
⋮	⋮
[R/S]	X6=30.0000
[R/S]	Y6=30.0000
[R/S]	END

User Instructions

SIZE: 010

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] GRIDALL	GRID ALL PTS
				N1 ?
3	Input the number of points:	N1	[R/S]	N2 ?
		N2	[R/S]	X1 ?
	the origin:	X1	[R/S]	Y1 ?
		Y1	[R/S]	H1 ?
	grid increments:	H1	[R/S]	H2 ?
		H2	[R/S]	THETA 1 ?
	and directions.	θ1	[R/S]	THETA 2 ?
	Calculate the coordinates of the grid	θ2	[R/S]	$X1=(x_1)$
	points.		[R/S]	$Y1=(y_1)$
			[R/S]	$X(i)=(x_i)$
			[R/S]	$Y(i)=(y_i)$
			⋮	⋮
			[R/S]	END
4	For another case, go to step 2.			

Program Listings

```
 01*LBL "GRI
DALL"
 02 1
 03 STO 09
 04 CF 29
 05 "GRID AL
L PTS"
 06 AVIEW
 07 PSE
 08 "N1 ?"
 09 PROMPT
 10 1
 11 -
 12 1 E3
 13 /
 14 STO 06
 15 STO 08
 16 "N2 ?"
 17 PROMPT
 18 1
 19 -
 20 1 E3
 21 /
 22 STO 07
 23 "X1 ?"
 24 PROMPT
 25 STO 00
 26 "Y1 ?"
 27 PROMPT
 28 STO 01
 29 "H1 ?"
 30 PROMPT
 31 +
 32 "H2 ?"
 33 PROMPT
 34 STO 04
 35 "THETA 1
 ?"
 36 PROMPT
 37 LASTX
 38 P-R
 39 STO 02
 40 RDN
 41 STO 03
 42 "THETA 2
 ?"
 43 PROMPT
 44 RCL 04
 45 P-R
 46 STO 04
 47 RDN
```

Initialize

Input N1, N2, X1, X2, H1, H2 θ1, θ2

Calculate ΔX's and ΔY's

```
 48 STO 05
 49 GTO d
50*LBL 01
 51 1
 52 ST+ 09
 53 ISG 06
 54 GTO d
 55 RCL 08
 56 STO 06
 57 ISG 07
 58 GTO d
 59 "END"
 60 AVIEW
 61 STOP
62*LBL d
 63 RCL 06
 64 INT
 65 RCL 02
 66 *
 67 RCL 04
 68 RCL 07
 69 INT
 70 *
 71 +
 72 RCL 00
 73 +
 74 "X"
 75 XEQ 12
 76 RCL 03
 77 RCL 06
 78 INT
 79 *
 80 RCL 05
 81 RCL 07
 82 INT
 83 *
 84 +
 85 RCL 01
 86 +
 87 "Y"
 88 XEQ 12
 89 GTO 01
90*LBL 12
 91 FIX 0
 92 ARCL 09
 93 "⊢="
 94 FIX 4
 95 ARCL X
 96 AVIEW
 97 STOP
 98 RTN
```

Loop control routine

Calculate X, Y

Display routine

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	X1	50	
	Y1		
	ΔX_1		
	ΔY_1		
	ΔX_2		
05	ΔY_2	55	
	$1.00N_1-1$		
	$1.00N_2-1$		
	$.00N_1-1$		
	Counter		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 010 TOT. REG. 37 USER MODE

ENG ____ FIX ____ SCI ____ ON X OFF ____

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
29	C	For proper display format	

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

GRID OF POINTS: CALCULATE DISCRETE POINTS

This program calculates the cartesian coordinates of specified points of a grid defined as follows:

a. First direction:

the angle θ_1 (related to positive X axis)

the distance between each point, H_1, in this direction

b. Second direction:

the angle θ_2

the distance H_2

c. Starting point (origin of the grid), X_{11} and Y_{11} .

Formulas:

$$Xij = X_1 + (j-1)\ \Delta X_1 + (i-1)\ \Delta X_2$$

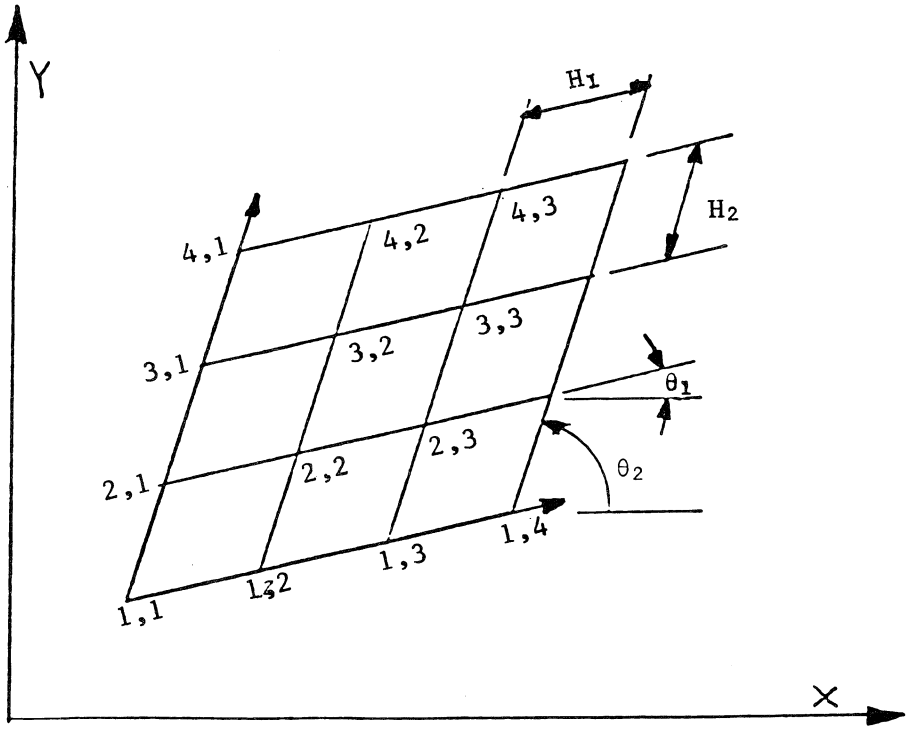
$$Yij = Y_1 + (j-1)\ \Delta Y_1 + (i-1)\ \Delta Y_2$$

where $\Delta X_1 = H_1 \cos \theta_1$

$\Delta Y_1 = H_1 \sin \theta_1$

$\Delta X_2 = H_2 \cos \theta_2$

$\Delta Y_2 = H_2 \sin \theta_2$



Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

For a grid with its origin at (1,1), $H_1 = 2$, $H_2 = 3$, $\theta_1 = 30$, and $\theta_2 = 90°$, find the cartesian coordinates for the following grid coordinates: (1,1), (2.5,4).

Keystrokes:	Display:
[USER]	(set USER mode)
[XEQ] [ALPHA] SIZE [ALPHA] 008	
[XEQ] [ALPHA] GRIDISC [ALPHA]	GRID DIS. PTS.
	X1 ?
1 [R/S]	Y1 ?
1 [R/S]	H1 ?
2 [R/S]	H2 ?
3 [R/S]	THETA 1 ?
30 [R/S]	THETA 2 ?
90 [R/S]	I?
1 [R/S]	J?
1 [R/S]	X=1.0000
[R/S]	Y=1.0000
[A]	I?
2.5 [R/S]	J?
4 [R/S]	X=6.1962
[R/S]	Y=8.5000

User Instructions

SIZE: 008

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program and set USER mode.		[USER]	
2	Initialize the program.		[XEQ] GRIDISC	GRID DIS. PTS.
3	Input grid parameters:			X1 ?
		x_1	[R/S]	Y1 ?
		y_1	[R/S]	H1 ?
		H_1	[R/S]	H2 ?
		H_2	[R/S]	THETA 1 ?
		θ_1	[R/S]	THETA 2 ?
		θ_2	[R/S]	I?
4	Input i, j, and calculate grid x,y			
	coordinates.	i	[R/S]	J?
		j	[R/S]	X=(x)
			[R/S]	Y=(y)
5	For additional points on the grid:		[A]	I?
	then go to step 4.			
6	For a new case, go to step 2.			

Program Listings

```
 01*LBL "GRI
DISC"
 02 "GRID DI
S. PTS."
 03 AVIEW
 04 PSE
 05 "X1 ?"
 06 PROMPT
 07 STO 00
 08 "Y1 ?"
 09 PROMPT
 10 STO 01
 11 "H1 ?"
 12 PROMPT
 13 +
 14 "H2 ?"
 15 PROMPT
 16 STO 04
 17 "THETA 1
 ?"
 18 PROMPT
 19 LASTX
 20 P-R
 21 STO 02
 22 RDN
 23 STO 03
 24 "THETA 2
 ?"
 25 PROMPT
 26 RCL 04
 27 P-R
 28 STO 04
 29 RDN
 30 STO 05
 31*LBL A
 32 "I?"
 33 PROMPT
 34 1
 35 -
 36 STO 06
 37 "J?"
 38 PROMPT
 39 1
 40 -
 41 STO 07
 42 RCL 02
 43 *
 44 RCL 04
 45 RCL 06
 46 *
 47 +
```

Initialize

Input x_1, y_2, H1, H2, , and calculate θx's and θy's

Input i, j and calculate x, y

```
 48 RCL 00
 49 +
 50 "X"
 51 XEQ 11
 52 RCL 03
 53 RCL 07
 54 *
 55 RCL 05
 56 RCL 06
 57 *
 58 +
 59 RCL 01
 60 +
 61 "Y"
 62*LBL 11
 63 "⊢="
 64 ARCL X
 65 AVIEW
 66 STOP
 67 RTN
 68 .END.
```

Display routine

80

90

00

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	X_1	50	
	Y_1		
	ΔX_1		
	ΔY_1		
	ΔX_2		
05	ΔY_2	55	
	I-1		
	J-1		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 008 TOT. REG. 27 USER MODE
ENG ____ FIX ____ SCI ____ ON X OFF ____
DEG X RAD ____ GRAD ____

FLAGS

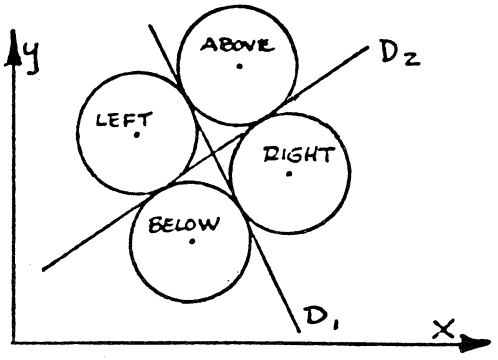
#	INIT S/C	SET INDICATES	CLEAR INDICATES

ASSIGNMENTS

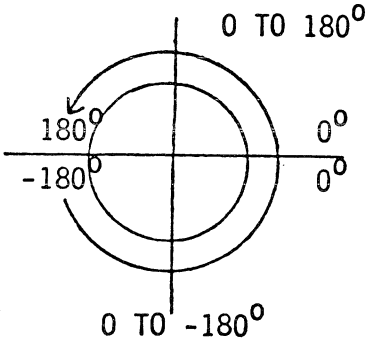
FUNCTION	KEY	FUNCTION	KEY

TANGENT CIRCLE TO TWO STRAIGHT LINES WITH A GIVEN RADIUS

This program calculates the X and Y coordinates of the centers of the four circles with a given radius, R, which are tangent to two given lines.



The straight lines are each defined by one point and an angle which follows the convention below:



The straight lines are first shifted by R. The calculation is then one of the intersection of two straight lines.

Formulas used:

$$X = \frac{(Y_2 - Y_1)\cos\theta_1\cos\theta_2 + X_1\sin\theta_1\cos\theta_2 - X_2\sin\theta_2\cos\theta_1}{\sin(\theta_1 - \theta_2)}$$

$$Y = Y_1 + (X - X_1)\tan\theta_1, \quad |\theta| \geq 90°$$

$$Y = Y_2 + (X - X_2)\tan\theta_2, \quad |\theta| < 90°$$

Reference: HP-67/97 "Geometry" Users' Library Solutions Book.

Example:

Find the tangent circle for:

$D_1 = [10,20,30°]$ $D_2 = [-20,30,-60°]$ $R = 10$

Executing the program four times will yield:

	X	Y
Above (A)	-4.5096	23.1699
Below (B)	-11.8301	-4.1506
Left (L)	-21.8301	13.1699
Right (R)	5.4904	5.8494

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 009	
[XEQ] [ALPHA] TANGENT [ALPHA]	TANGENT CIRC.
	R?
10 [R/S]	WHERE(L,R,A,B) ?
A [R/S]	X?
10 [R/S]	Y?
20 [R/S]	THETA?
30 [R/S]	X?
20 [CHS] [R/S]	Y?
30 [R/S]	THETA ?
60 [CHS] [R/S]	X=-4.5096
[R/S]	Y=23.1699

User Instructions

SIZE: 009

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program.			
2	Initialize the program.		[XEQ] TANGENT	TANGENT CIRC.
				R?
3	Input R.	R	[R/S]	WHERE (L,R,A,B)?
4	Input position L = left; R = right;			
	A = above; B = below.	pos	[R/S]	X?
5	Input X_i, Y_i, θ_i.	X_1	[R/S]	Y?
		Y_1	[R/S]	THETA ?
		θ_1	[R/S]	X?
		X_2	[R/S]	Y?
		Y_2	[R/S]	THETA ?
		θ_2	[R/S]	X=(X)
			[R/S]	Y=(Y)
6	For another case, go to step 2.			

Program Listings

```
 01*LBL "TAN
GENT"
 02 CF 01
 03 CF 02
 04 CF 03
 05 CF 04
 06 "TANGENT
 CIRC."
 07 AVIEW
 08 PSE
 09 "R?"
 10 PROMPT
 11 STO 08
 12 "WHERE<L
,R,A,B>?"
 13 AON
 14 PROMPT
 15 AOFF
 16 ASTO Y
 17 "A"
 18 ASTO X
 19 X=Y?
 20 SF 02
 21 "B"
 22 ASTO X
 23 X=Y?
 24 SF 03
 25 "L"
 26 ASTO X
 27 X=Y?
 28 SF 04
 29*LBL 07
 30 "X?"
 31 PROMPT
 32 "Y?"
 33 PROMPT
 34 "THETA ?
"
 35 PROMPT
 36 FS? 02
 37 GTO B
 38 FS? 03
 39 GTO C
 40 FS? 04
 41 GTO D
 42 X<0?
 43 GTO B
 44 GTO C
 45*LBL D
 46 X<0?
 47 GTO C
```

Initialize

Input R

Input position

Input X, Y, θ

Set up calculations

```
 48*LBL B
 49 RCL 08
 50 GTO 01
 51*LBL C
 52 RCL 08
 53 CHS
 54*LBL 01
 55 X<>Y
 56 FS?C 01
 57 GTO 02
 58 STO 03
 59 X<>Y
 60 P-R
 61 X<>Y
 62 RDN
 63 +
 64 STO 02
 65 RDN
 66 X<>Y
 67 -
 68 STO 01
 69 SF 01
 70 GTO 07
 71*LBL 02
 72 STO 06
 73 X<>Y
 74 P-R
 75 X<>Y
 76 RDN
 77 +
 78 STO 05
 79 RDN
 80 X<>Y
 81 -
 82 STO 04
 83 RCL 05
 84 RCL 02
 85 -
 86 RCL 03
 87 COS
 88 STO 07
 89 *
 90 RCL 06
 91 COS
 92 STO 08
 93 *
 94 RCL 01
 95 RCL 08
 96 *
 97 RCL 03
 98 SIN
```

Calculate X_1, Y_1

Calculate X, Y

Calculate X

Program Listings

```
 99 *
100 +
101 RCL 04
102 RCL 07
103 *
104 RCL 06
105 SIN
106 *
107 -
108 RCL 03
109 RCL 06
110 -
111 SIN
112 /
113 STO 07
114 "X"
115 XEQ 11
116 RCL 03
117 ABS
118 90
119 X>Y?
120 GTO 03
121 RCL 07
122 RCL 01
123 -
124 RCL 03
125 TAN
126 *
127 RCL 02
128 GTO 04
129◆LBL 03
130 RCL 07
131 RCL 04
132 -
133 RCL 06
134 TAN
135 *
136 RCL 05
137◆LBL 04
138 +
139 "Y"
140◆LBL 11
141 "⊢="
142 ARCL X
143 AVIEW
144 STOP
145 RTN
146 .END.
```

Calculate Y

Display routine

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00		50	
	X'_1		
	Y'_1		
	θ_1		
	X'_2		
05	Y'_2	55	
	θ_2		
	$\cos\theta_1$, X		
	R, $\cos\theta_2$		
10		60	
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 009 TOT. REG. 44 USER MODE
ENG ___ FIX ___ SCI ___ ON ___ OFF X
DEG X RAD ___ GRAD ___

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
01		1st pass, input X_1	2nd pass, input X_2,...
02		circle above	
03		circle below	
04		circle left	

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

DISTANCE BETWEEN LINES IN SPACE

Given two lines, each defined by two points, this program calculates the shortest distance between the two lines. (This program was originally written to determine the clearance between electrical distribution circuits and guy wires or supporting structures).

The program takes lines defined by the two-point form,

$$\frac{x - x_1}{x_1' - x_1} = \frac{y - y_1}{y_1' - y_1} = \frac{z - z_1}{z_1' - z_1}$$

changes them to the point-direction form,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

and the shortest distance (D) is calculated by:

$$D = \pm \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}^2 + \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2 + \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2}}$$

Reference: Handbook of Tables for Mathematics, Third Edition, Samuel M. Selby, Published by The Chemical Rubber Co., 1967, page 509.

Example:

Given two lines in three-dimensional space:

Line #1 defined by points $(X_1,Y_1,Z_1) = (30,14,10)$ and $(X'_1,Y'_1,Z'_1) = (0,46,10)$;
Line #2 defined by points $(X_2,Y_2,Z_2) = (124,50,-30)$ and $(X'_2,Y'_2,Z'_2) = (0,36,16)$.

Calculate the shortest distance between the two lines.

Keystrokes:	Display:
[XEQ] [ALPHA] SIZE [ALPHA] 014	
[XEQ] [ALPHA] DIST [ALPHA]	DIST. B. LINES
	X1 ?
30 [R/S]	Y1 ?
14 [R/S]	Z1 ?
10 [R/S]	X1-PRIME ?
0 [R/S]	Y1-PRIME ?
46 [R/S]	Z1-PRIME ?
10 [R/S]	X2 ?
124 [R/S]	Y2 ?
50 [R/S]	Z2 ?
30 [CHS] [R/S]	X2-PRIME ?
0 [R/S]	Y2-PRIME ?
36 [R/S]	Z2-PRIME ?
16 [R/S]	D=2.5940

User Instructions

SIZE: 014

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Key in the program.			
2	Initialize the program.		[XEQ] DIST	DIST. B.LINES
3	Input line parameters:			X1 ?
		x_1	[R/S]	Y1 ?
		y_1	[R/S]	Z1 ?
		z_1	[R/S]	X1-PRIME ?
		x'_1	[R/S]	Y1-PRIME ?
		y'_1	[R/S]	Z1-PRIME ?
		z'_1	[R/S]	X2 ?
		x_2	[R/S]	Y2 ?
		y_2	[R/S]	Z2 ?
		z_2	[R/S]	X2-PRIME ?
		x'_2	[R/S]	Y2-PRIME ?
		y'_2	[R/S]	Z2-PRIME ?
	and calculate D.	z'_2	[R/S]	D=(D)*
4	For a new case, go to step 2.			
	*Negative sign, if present, has no meaning.			

Program Listings

```
01*LBL "DIS
T"
02 "DIST. B
. LINES"
03 AVIEW
04 2.2
05 STO 00
06 1.002
07 STO 01
08 CF 29
09 FIX 0
10*LBL 02
11 "X"
12 XEQ 12
13 "Y"
14 XEQ 12
15 "Z"
16 XEQ 12
17 "X"
18 XEQ 13
19 "Y"
20 XEQ 13
21 "Z"
22 XEQ 13
23 ISG 01
24 GTO 02
25 RCL 08
26 ST- 11
27 RCL 09
28 ST- 12
29 RCL 10
30 ST- 13
31 RCL 02
32 ST- 05
33 ST- 08
34 RCL 03
35 ST- 06
36 ST- 09
37 RCL 04
38 ST- 07
39 ST- 10
40 RCL 11
41 RCL 06
42 RCL 05
43 RCL 12
44 XEQ 14
45 RCL 12
46 RCL 07
47 RCL 06
48 RCL 13
49 XEQ 14
```

Initialize

Input data

Calculate a_i, b_i, c_i, ΔX, ΔY, ΔZ

Calculate (A-B), (B-C) (C-A)

```
50 RCL 13
51 RCL 05
52 RCL 07
53 RCL 11
54 XEQ 14
55 RCL 05
56 X↑2
57 RCL 03
58 X↑2
59 +
60 RCL 04
61 X↑2
62 +
63 SQRT
64 1/X
65 RCL 08
66 RCL 04
67 *
68 RCL 09
69 RCL 05
70 *
71 +
72 RCL 10
73 RCL 03
74 *
75 +
76 *
77 FIX 4
78 "D="
79 ARCL X
80 AVIEW
81 STOP
82*LBL 13
83 ARCL 01
84 "⊢-PRIME
?"
85 GTO 15
86*LBL 12
87 ARCL 01
88 "⊢ ?"
89*LBL 15
90 PROMPT
91 STO IND
00
92 ISG 00
93 RTN
94*LBL 14
95 *
96 STO IND
01
97 RDN
```

Calculate D

Input prompting routine

Calculate 2x2 matrix

Program Listings

```
 98 *
 99 ST- IND
01
100 ISG 01
101 RTN
102 .END.
```

REGISTERS, STATUS, FLAGS, ASSIGNMENTS

DATA REGISTERS

00	pointer	50	
	counter		
	x_1		
	y_1, (A-B)		
	z_1, (B-C)		
05	x'_1, a_1, (C-A)	55	
	y'_1, b_1		
	z'_1, c_1		
	x_2, x_2-x_1		
	y_2, y_2-y_1		
10	z_2, z_2-z_1	60	
	x'_2, a_2		
	y'_2, b_2		
	z'_2, c_2		
15		65	
20		70	
25		75	
30		80	
35		85	
40		90	
45		95	

STATUS

SIZE 014 TOT. REG. 41 USER MODE

ENG ____ FIX ____ SCI ____ ON ____ OFF X

DEG X RAD ____ GRAD ____

FLAGS

#	INIT S/C	SET INDICATES	CLEAR INDICATES
29	C	For proper display format	

ASSIGNMENTS

FUNCTION	KEY	FUNCTION	KEY

NOTES

HEWLETT-PACKARD

HP-41C

USERS' LIBRARY SOLUTIONS

Bar Codes

Geometry

GEOMETRY

SINE PLATE SOL/COORD OF POINT
POS. & SLOPE OF INCLINED HOLE
PROGRAM REGISTERS NEEDED: 27

HEWLETT PACKARD
SOLUTION BOOK:
GEOMETRY

ROW 1 (1 - 2)

ROW 2 (2 - 6)

ROW 3 (7 - 15)

ROW 4 (16 - 26)

ROW 5 (27 - 38)

ROW 6 (39 - 44)

ROW 7 (44 - 50)

ROW 8 (51 - 59)

ROW 9 (60 - 66)

ROW 10 (66 - 76)

ROW 11 (76 - 82)

ROW 12 (83 - 87)

ROW 13 (87 - 92)

ROW 14 (93 - 100)

ROW 15 (100 - 100)

ROW 1 (1 - 3)

ROW 2 (3 - 3)

ROW 3 (3 - 9)

ROW 4 (10 - 18)

ROW 5 (18 - 29)

ROW 6 (30 - 38)

ROW 7 (38 - 44)

ROW 8 (45 - 53)

ROW 9 (54 - 62)

ROW 10 (63 - 75)

ROW 11 (76 - 88)

ROW 12 (88 - 98)

ROW 13 (99 - 105)

ROW 14 (105 - 108)

ROW 15 (108 - 111)

INTERNAL AND EXTERNAL TAPERS

PROGRAM REGISTERS NEEDED: 37

HEWLETT PACKARD
SOLUTION BOOK:
GEOMETRY

ROW 1 (1 - 3)

ROW 2 (3 - 3)

ROW 3 (3 - 10)

ROW 4 (10 - 19)

ROW 5 (20 - 30)

ROW 6 (31 - 40)

ROW 7 (41 - 49)

ROW 8 (50 - 55)

ROW 9 (55 - 62)

ROW 10 (62 - 69)

ROW 11 (69 - 76)

ROW 12 (76 - 84)

ROW 13 (85 - 96)

ROW 14 (97 - 105)

ROW 15 (105 - 114)

ROW 16 (115 - 125)

ROW 17 (125 - 133)

ROW 18 (133 - 138)

ROW 19 (139 - 150)

ROW 20 (150 - 155)

POINTS OF TANGENCY WITH
CIRCLES AND ARCS
PROGRAM REGISTERS NEEDED: 39

HEWLETT PACKARD
SOLUTION BOOK:
GEOMETRY

ROW 1 (1 - 2)

ROW 2 (2 - 5)

ROW 3 (5 - 13)

ROW 4 (14 - 26)

ROW 5 (26 - 32)

ROW 6 (33 - 37)

ROW 7 (37 - 44)

ROW 8 (45 - 55)

ROW 9 (56 - 67)

ROW 10 (68 - 76)

ROW 11 (77 - 82)

ROW 12 (83 - 87)

ROW 13 (87 - 91)

ROW 14 (92 - 99)

ROW 15 (100 - 109)

ROW 16 (110 - 120)

ROW 17 (120 - 127)

ROW 18 (128 - 134)

ROW 19 (134 - 139)

ROW 20 (139 - 144)

ROW 21 (144 - 149)

LINE–LINE INTERSECTION

PROGRAM REGISTERS NEEDED: 26

ROW 1 (1 – 3)

ROW 2 (3 – 5)

ROW 3 (5 – 12)

ROW 4 (12 – 17)

ROW 5 (18 – 24)

ROW 6 (24 – 31)

ROW 7 (31 – 37)

ROW 8 (37 – 42)

ROW 9 (42 – 46)

ROW 10 (47 – 49)

ROW 11 (49 – 56)

ROW 12 (57 – 68)

ROW 13 (69 – 77)

ROW 14 (78 – 84)

POINTS ON A STRAIGHT LINE

PROGRAM REGISTERS NEEDED: 15

ROW 1 (1 - 2)

ROW 2 (2 - 4)

ROW 3 (5 - 9)

ROW 4 (10 - 13)

ROW 5 (13 - 22)

ROW 6 (23 - 32)

ROW 7 (33 - 41)

ROW 8 (42 - 48)

GRID OF POINTS:
CALCULATE ALL POINTS
PROGRAM REGISTERS NEEDED: 28

HEWLETT PACKARD
SOLUTION BOOK:
GEOMETRY

ROW 1 (1 - 3)

ROW 2 (4 - 5)

ROW 3 (5 - 12)

ROW 4 (12 - 19)

ROW 5 (20 - 26)

ROW 6 (26 - 31)

ROW 7 (32 - 35)

ROW 8 (35 - 42)

ROW 9 (42 - 48)

ROW 10 (49 - 55)

ROW 11 (56 - 62)

ROW 12 (62 - 74)

ROW 13 (74 - 84)

ROW 14 (85 - 92)

ROW 15 (92 - 99)

ROW 16 (99 - 99)

GRID OF POINTS:
CALCULATE DISCRETE POINTS
PROGRAM REGISTERS NEEDED: 20

HEWLETT PACKARD
SOLUTION BOOK:
GEOMETRY

ROW 1 (1 - 2)

ROW 2 (2 - 2)

ROW 3 (3 - 8)

ROW 4 (8 - 14)

ROW 5 (14 - 17)

ROW 6 (17 - 24)

ROW 7 (24 - 32)

ROW 8 (32 - 41)

ROW 9 (42 - 51)

ROW 10 (52 - 63)

ROW 11 (63 - 68)

TANGENT CIRCLE TO TWO STRAIGHT LINES WITH A GIVEN RADIUS
PROGRAM REGISTERS NEEDED: 36

ROW 1 (1 – 2)

ROW 2 (3 – 6)

ROW 3 (6 – 10)

ROW 4 (11 – 12)

ROW 5 (12 – 18)

ROW 6 (19 – 26)

ROW 7 (26 – 33)

ROW 8 (34 – 37)

ROW 9 (37 – 43)

ROW 10 (43 – 48)

ROW 11 (49 – 57)

ROW 12 (58 – 69)

ROW 13 (70 – 81)

ROW 14 (82 – 94)

ROW 15 (95 – 107)

ROW 16 (108 – 117)

ROW 17 (118 – 128)

ROW 18 (128 – 139)

ROW 19 (140 – 146)

DISTANCE BETWEEN
LINES IN SPACE
PROGRAM REGISTERS NEEDED: 29

HEWLETT PACKARD
SOLUTION BOOK:
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ROW 1 (1 - 2)

ROW 2 (2 - 4)

ROW 3 (4 - 10)

ROW 4 (11 - 16)

ROW 5 (16 - 21)

ROW 6 (21 - 28)

ROW 7 (28 - 36)

ROW 8 (36 - 44)

ROW 9 (45 - 54)

ROW 10 (54 - 66)

ROW 11 (67 - 78)

ROW 12 (78 - 84)

ROW 13 (84 - 88)

ROW 14 (88 - 95)

ROW 15 (96 - 102)

ROW 16 (102 - 102)

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Games
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Optometry II (Contact Lens)
Physics
Surveying

* Some books require additional memory modules to accomodate all programs.

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GRID OF POINTS: CALCULATE ALL POINTS
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DISTANCE BETWEEN LINES IN SPACE

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